A VARIETY OF CP VIOLATING B DECAYS

Michael Gronau

Department of Physics, Technion – Israel Institute of Technology

32000 Haifa, Israel

Abstract

A variety of CP violating asymmetries in B^0 decays is predicted in the Standard Model in terms of CKM (Cabibbo-Kobayashi-Maskawa) complex phases. These phases can also be determined from rate measurements of certain B^+ and B^- decays. We focus on the following processes: $B^0 \to \psi K_S$, $\pi^+\pi^-$, $\pi^0\pi^0$, $\rho^\pm\pi^\mp$, $B_s \to D_s^+ K^-$, $\pi^0\eta$, $B^+ \to D^0(\overline{D}^0)K^+$, $\pi^+\pi^0$, $\pi^0 K^+$, $\pi^+ K^0$, ηK^+ and their charge-conjugates. Complications due to gluonic-penguin and electroweak-penguin amplitudes are dealt with and are resolved. The importance of final state interaction phases in charged B decay asymmetries is demonstrated in $B^+ \to \chi_{c0}\pi^+$, $\chi_{c0} \to \pi^+\pi^-$ through the effect of the χ_{c0} width.

1. Introduction

The Standard Model [1] accounts for the observed CP violation in the neutral K meson mixing [2] through a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3]. The B meson system provides a wide variety of independent CP asymmetry measurements, which can test the hypothesis that this phase is the only source of CP violation in the presently known fundamental interactions among elementary particles. The purpose of this talk is to demonstrate some of the most promising ways of carrying out such tests. Since new ideas, theoretical as well as experimental, are constantly being developed in this field, I decided to combine in this talk some early and already "classical" examples with very recent suggestions. For earlier reviews of this subject, describing other processes and containing a more complete list of references, see ref. [4].

Section 2 introduces the CKM matrix and summarizes the available information on the magnitudes and phases of its elements. The subsequent two sections deal with neutral and with charged B decays. Section 3 describes CP violation which occurs when mixed neutral B mesons decay to states which are common decay products of B^0 and $\overline{B}^0$. Direct CP violation in charged B decays is the subject of Section 4, while Section 5 concludes.

2. CP violation in the standard model

In the standard model of three families of quarks and leptons the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is spontaneously broken by the vacuum expectation value of a single scalar Higgs doublet. CP violation occurs in the interactions of the three families of the left-handed quarks with the charged gauge boson:

$$-\mathcal{L} = (\overline{u} \quad \overline{c} \quad \overline{t}) \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (\overline{d} \quad \overline{s} \quad \overline{b}) \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$+ \frac{g}{\sqrt{2}} (\overline{u} \quad \overline{c} \quad \overline{t})_L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + ... \tag{1}$$

The unitary CKM mixing matrix V is defined [5] in terms of three Euler-like family-mixing angles , θ_{ij}, and a phase $\gamma \equiv \delta_{13}$ which is the mere origin of CP violation. The measured values of the three mixing angles, which have a hierarchical pattern, are [6]:

$$\sin \theta_{12} \equiv |V_{us}| = 0.220 \pm 0.002 \ ,$$

$$\sin \theta_{23} \equiv |V_{cb}| = 0.039 \pm 0.005 \ ,$$

$$\sin \theta_{13} \equiv |V_{ub}| = 0.0035 \pm 0.0015 \ . \tag{2}$$

The only information about a nonzero value of $\gamma \equiv \mathrm{Arg}(V_{ub}^*)$ comes from CP violation in the $K^0 - \overline{K}^0$ system, which provides very loose bounds on this phase (see eq. (4) below).

Unitarity of V implies triangle relations such as

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 , \tag{3}$$

which is shown in Fig. 1. The three angles of the unitarity triangle, α, β and γ are rather badly known at present. Current constarints, which depend on uncertainties in K- and B-meson hadronic parameters, can be summarized by the following ranges [6]:

$$10^0 \leq \alpha \leq 150^0, \quad 5^0 \leq \beta \leq 45^0, \quad 20^0 \leq \gamma \leq 165^0 . \tag{4}$$

As we will show in the following two sections, measurement of certain CP asymmetries in B decays are directly related to these three angles in a manner which is free of hadronic uncertainties.

We note in passing that in the standard phase convention [5] only two of the CKM elements, V_{ub} with phase $-\gamma$, and V_{td} with phase $-\beta$, carry a complex phase. All other matrix elements are real to a good approximation.

3. CP violation in decays of mixed $B^0 - \overline{B}^0$

3.1 Decays to CP eigenstates dominated by a single CKM phase [7]

Consider the time-evolution of a state which is identified ("tagged") as a B^0 at time $t = 0$:

$$t = 0: \quad |B^0\rangle = \frac{e^{-i\phi_M}}{\sqrt{2}}(|B_L\rangle + |B_H\rangle) . \tag{5}$$

$B_{L,H}$ are the "light" and "heavy" mass-eigenstates and ϕ_M is the phase of the $B^0 - \overline{B}^0$ mixing parameter [4], $\phi_M = \beta$ for B^0 and $\phi_M = 0$ for B_s. The $B_{L,H}$ state-evolutions are given by their masses and by their approximately equal decay width Γ: $|B_{L,H}(t = 0)\rangle \rightarrow |B_{L,H}(t)\rangle = \exp[-i(m_{L,H} - \frac{i}{2}\Gamma)t]|B_{L,H}(t = 0)\rangle$. Thus, the B^0 oscillates into a mixture of B^0 and $\overline{B}^0$:

$$t: \quad |B^0(t)\rangle = e^{-i\overline{m}t}e^{-\frac{\Gamma}{2}t}[\cos(\frac{\Delta mt}{2})|B^0\rangle + ie^{-2i\phi_M}\sin(\frac{\Delta mt}{2})|\overline{B}^0\rangle] , \tag{6}$$

where $\overline{m} \equiv (m_H + m_L)/2$, $\Delta m \equiv m_H - m_L$.

Let us consider decays into states $|f\rangle$ which are eigenstates of CP, $CP|f\rangle = \xi|f\rangle$ with eigenvalue $\xi = \pm 1$, and let us assume that a single weak amplitude (or rather a single weak phase) dominates the decay process. Both B^0 and $\overline{B}^0$ decay to the state f, with amplitudes $A = |A|\exp(i\phi_f)\exp(i\delta_f)$ and $\overline{A} = \xi|A|\exp(-i\phi_f)\exp(i\delta_f)$, respectively.

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ϕ_f and δ_f are the weak and strong phases, respectively. The former changes sign under charge-conjugation, whereas the latter remains the same. The time-dependent decay rate of an initial B^0 is

$$\Gamma(t) \equiv \Gamma(B^0(t) \to f) = e^{-\Gamma t}|A|^2[1 + \xi \sin 2(\beta + \phi_f) \sin(\Delta mt)] , \qquad (7)$$

and the corresponding rate for an initial $\overline{B}^0$ is

$$\overline{\Gamma}(t) \equiv \Gamma(\overline{B}^0(t) \to f) = e^{-\Gamma t}|A|^2[1 - \xi \sin 2(\beta + \phi_f) \sin(\Delta mt)] . \qquad (8)$$

The time-dependent CP asymmetry is then given by

$$Asym.(t) \equiv \frac{\Gamma(t) - \overline{\Gamma}(t)}{\Gamma(t) + \overline{\Gamma}(t)} = \xi \sin 2(\phi_M + \phi_f) \sin(\Delta mt) , \qquad (9)$$

while the time-integrated asymmetry is

$$Asym. = \xi \Big(\frac{\Delta m/\Gamma}{1 + (\Delta m/\Gamma)^2}\Big) \sin 2(\phi_M + \phi_f) . \qquad (10)$$

That is, in this case *the CP asymmetry measures a CKM phase with no hadronic uncertainty.* The integrated asymmetry in B^0 decays may be as large as $(\Delta m/\Gamma)/[1 + (\Delta m/\Gamma)^2] = 0.47$.

The best example of decays to CP-eigenstates is the well-known gold-plated case of $B^0 \to \psi K_S$, for which a branching ratio of about 5×10^{-4} has already been measured [8]. In this case $\phi_M = \beta$, $\phi_f = \text{Arg}(V_{cb}^* V_{cs}) = 0$, $\xi = -1$. Another case is $B^0 \to \pi^+\pi^-$, for which a combined branching ratio $B(B^0 \to \pi^+\pi^-$ and $K^+\pi^-) = (1.8 \pm 0.6) \times 10^{-5}$ has been measured [9], with a likely solution in which the two modes have about equal branching ratios. In this case $\phi_f = \text{Arg}(V_{ub}^* V_{ud}) = \gamma$, $\xi = 1$. Consequently one has in these two cases

$$Asym.(B^0 \to \psi K_S; t) = -\sin 2\beta \sin(\Delta mt) ,$$

$$Asym.(B^0 \to \pi^+\pi^-; t) = -\sin 2\alpha \sin(\Delta mt) . \qquad (11)$$

In the case of decay to two pions the asymmetry obtains, however, corrections from a second (penguin) CKM phase. This problem will be discussed below.

3.2 Decays to non-CP eigenstates [10][11]

Angles of the unitarity triangle can also be determined from neutral B decays to states f which are not eigenstates of CP. This is feasible when both a B^0 and a $\overline{B}^0$ can decay to a final state which appears in only one partial wave, provided that a single CKM phase dominates each of the corresponding decay amplitudes.

The time-dependent rates for states which are B^0 or $\overline{B}^0$ at $t = 0$ and decay at time t to a state f or its charge-conjugate $\overline{f}$ are given by:

$$\Gamma_f(t) = e^{-\Gamma t}[|A|^2 \cos^2(\frac{\Delta mt}{2}) + |\overline{A}|^2 \sin^2(\frac{\Delta mt}{2}) + |A\overline{A}| \sin(\Delta\delta + \Delta\phi_f + 2\phi_M) \sin(\Delta mt)] \ ,$$

$$\overline{\Gamma}_f(t) = e^{-\Gamma t}[|\overline{A}|^2 \cos^2(\frac{\Delta mt}{2}) + |A|^2 \sin^2(\frac{\Delta mt}{2}) - |A\overline{A}| \sin(\Delta\delta + \Delta\phi_f + 2\phi_M) \sin(\Delta mt)] \ ,$$

$$\Gamma_{\overline{f}}(t) = e^{-\Gamma t}[|\overline{A}|^2 \cos^2(\frac{\Delta mt}{2}) + |A|^2 \sin^2(\frac{\Delta mt}{2}) - |A\overline{A}| \sin(\Delta\delta - \Delta\phi_f - 2\phi_M) \sin(\Delta mt)] \ ,$$

$$\overline{\Gamma}_{\overline{f}}(t) = e^{-\Gamma t}[|A|^2 \cos^2(\frac{\Delta mt}{2}) + |\overline{A}|^2 \sin^2(\frac{\Delta mt}{2}) + |A\overline{A}| \sin(\Delta\delta - \Delta\phi_f - 2\phi_M) \sin(\Delta mt)] \ .$$

$$(12)$$

Here $\Delta\delta_f$, $(\Delta\phi_f)$ is the difference between the strong (weak) phases of A and $\overline{A}$, the decay amplitudes of B^0 and $\overline{B}^0$ to the common state f. The four rates depend on four unknown quantities, $|A|$, $|\overline{A}|$, $\sin(\Delta\delta_f + \Delta\phi_f + 2\phi_M)$, $\sin(\Delta\delta_f - \Delta\phi_f - 2\phi_M)$. Measurement of the rates allows a determination of the weak CKM phase $\Delta\phi_f + 2\phi_M$ apart from a two-fold ambiguity.

There are at least two interesting examples to which this method has been applied [11]. In the first case, $B^0 \to \rho^+\pi^-$, one must neglect a second contribution of a penguin amplitude, a problem which will be addressed in the following two subsections. Assuming for a moment that tree diagrams dominate A and $\overline{A}$, one can measure in this manner the angle α. A second case, which may be used to measure γ, is $B_s \to D_s^+ K^-$.

3.3 Corrections from penguin amplitudes [12]

In a wide variety of decay processes, such as in $B^0 \to \pi^+\pi^-$, there exists a second amplitude due to a "penguin" diagram in addition to the usual "tree" diagram. The two diagrams cary different CKM phases and in general may have different strong phases. In such a case CP is violated in the direct decay of a B^0. Then in decays to CP-eigenstates one has $|A| \neq |\overline{A}|$, and the asymmetry acquires a time-dependent cosine term in addition to the sine term:

$$Asym.(t) = \frac{(1 - |\overline{A}/A|^2)\cos(\Delta mt) - 2\mathrm{Im}(e^{-2i\phi_M}\overline{A}/A)\sin(\Delta mt)}{1 + |\overline{A}/A|^2} \ . \qquad (13)$$

Observation of an extra $\cos(\Delta mt)$ term implies direct CP violation and would invalidate the above method of measuring a weak phase. The opposite is not true, however, since the absence of a cosine term does not guarantee that the coefficient of the sine term is given in terms of a CKM phase. The coefficient of $\cos(\Delta mt)$ is proportional to $\sin(\Delta\delta)$, where $\Delta\delta$ is the final-state phase-difference between the tree and penguin amplitudes. On

the other hand, the coefficient of $\sin(\Delta mt)$ obtains a correction proportional to $\cos(\Delta\delta)$. If $\Delta\delta$ were small this correction might be large, in spite of the fact that the $\cos(\Delta mt)$ term were too small to be observed. Assuming, for instance $\Delta\delta = 0$ in $B^0 \to \pi^+\pi^-$, where the penguin-to-tree ratio of amplitudes may be as large as about 0.2, the coefficient of $\sin(\Delta mt)$ may be as large as 0.4 for $\sin(2\alpha) = 0$ [13] (for which no asymmetry is expected in the absence of penguins). On the other hand, the decay $B^0 \to \psi K_S$ remains a pure case, since in this case the penguin-to-tree ratio of amplitudes with unequal weak phases is extremely small.

3.4 Removing penguin corrections in $B^0 \to \pi^+\pi^-$ [14]

It is possible to disentangle the penguin contribution in $B^0 \to \pi^+\pi^-$ from the tree-dominating asymmetry by measuring also the rates of $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$. No time-dependence is required for these processes. The method is based on the observation that the two weak operators contributing to these decays have different isospin properties. Whereas the tree operator is a mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$, the gluonic penguin operator is pure $\Delta I = 1/2$.

The physical amplitudes of $B \to \pi^+\pi^-, \pi^0\pi^0, \pi^+\pi^0$ can be decomposed into their isospin components

$$\frac{1}{\sqrt{2}}A^{+-} = A_2 - A_0 , \quad A^{00} = 2A_2 + A_0 , \quad A^{+0} = 3A_2 , \tag{14}$$

where A_0 and A_2 correspond to $\pi\pi$ states with $I = 0$ and $I = 2$, respectively. This yields a complex triangle relation

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0} , \tag{15}$$

and a similar triangle relation for the charge-conjugated processes

$$\frac{1}{\sqrt{2}}\overline{A}^{+-} + \overline{A}^{00} = \overline{A}^{-0} . \tag{16}$$

Applying a simple phase rotation, $\tilde{A} = \exp(2i\gamma)\overline{A}$ (γ being the phase of the tree amplitude), the two triangles (15)(16) are described in Fig. 2, sharing a common base, $|A^{+0}| = |\tilde{A}^{-0}|$ (at this point we neglect the electroweak penguin contributions P_{EW}, $\tilde{P}_{EW}$). The angle θ between the sides of the two triangles corresponding to A^{+-} and $\tilde{A}^{+-}$ and the ratio of the lengths of these sides determine, for a given α, the coefficient of the $\sin(\Delta mt)$ term in $B^0(t) \to \pi^+\pi^-$:

$$\text{coefficient of } \sin(\Delta mt) = \frac{|\tilde{A}^{+-}|}{|A^{+-}|}\sin(2\alpha + \theta) . \tag{17}$$

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Thus, the time-dependence of $B^0(t) \to \pi^+\pi^-$, and the integrated decay rates involving neutral pions, determine α. There exists a two-old ambiguity in determining α due to the fact that one of the two triangles in Fig. 2 may be turned up-side-down

Recently it was noted [15] that electroweak penguin contributions could spoil this method, since these amplitudes are not pure $\Delta I = 1/2$. A closer look at the effect shows that, in fact, the uncertainty in determining α remains very small [16]. The effect is shown in Fig. 2, where the terms P_{EW} and $\tilde{P}_{EW}$ represent the electroweak penguin contributions to B and $\overline{B}$ decays, respectively. As a result of these terms the two triangles, for B and $\overline{B}$ decays, do not share a common base. CP is violated also in $B^+ \to \pi^+\pi^0$. This introduces a small uncertainly in determining θ and subsequently α. Model-dependent calculations [15] and more general order-of-magnitude arguments [16] show the following hierarchy among the tree (T), gluonic penguin (P) and electroweak penguin (P_{EW}) contributions in $B \to \pi\pi$

$$T : P : P_{EW} \sim 1 : \lambda : \lambda^2 , \quad \lambda = 0.2 . \tag{18}$$

Therefore, the uncertainty in α from electroweak penguin amplitudes is at most a few degrees:

$$\Delta\alpha = \frac{1}{2}\Delta\theta \leq \lambda^2 . \tag{19}$$

A similar isospin analysis can be carried out for $B \to \rho\pi$ [17]. To resolve certain ambiguities in α, a full Dalitz plot analysis must be made for the three pion final states.

4. CP violation in charged B decays

4.1 The problem

The simplest manifestation of CP violation, which requires neither tagging nor a time-dependent measurement, is finding different partial decay widths for a particle and its antiparticle into corresponding decay modes. Consider a general decay $B^+ \to f$ and its charge-conjugate process $B^- \to \overline{f}$. In order that these two proceses have different rates, two amplitudes (A_1, A_2) must contribute, with different CKM phases $(\phi_1 \neq \phi_2)$ and different final state interaction phases $(\delta_1 \neq \delta_2)$:

$$A(B^+ \to f) = |A_1|e^{i\phi_1}e^{i\delta_1} + |A_2|e^{i\phi_2}e^{i\delta_2} ,$$

$$\overline{A}(B^- \to \overline{f}) = |A_1|e^{-i\phi_1}e^{i\delta_1} + |A_2|e^{-i\phi_2}e^{i\delta_2} ,$$

$$|A|^2 - |\overline{A}|^2 = 2|A_1 A_2| \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) . \tag{20}$$

For $|A_2|^2 \ll |A_1|^2$ one finds a CP asymmetry

$$Asym. = \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} \approx 2\frac{|A_2|}{|A_1|} \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2) . \tag{21}$$

The theoretical difficulty of relating an asymmetry in charged B decays to a pure CKM phase, $(\phi_1 - \phi_2)$, follows from having two unknowns in the problem: The ratio of amplitudes, $|A_2/A_1|$, and the final state phase difference, $\delta_2 - \delta_1$. Both quantities involve quite large theorertical uncertainties. In this section I will discuss three kinds of solutions to this problem:

  a. Measure A_1 and A_2 independently (4.2).

  b. Relate A_1 and A_2 by symmetry to other directly measurable amplitudes (4.3).

  c. Control $\delta_2 - \delta_1$ as much as possible (4.4).

4.2 Measuring γ in $B^\pm \to D^0 K^\pm$ [18]

The decays $B^\pm \to D_1^0(D_2^0)K^\pm$ and a few other processes of this type provide a unique case, in which one can measure separately the magnitudes of the two contributing amplitudes, and thereby determine the CKM phase γ.

$D_1^0(D_2^0) = (D^0 + (-)\overline{D}^0)/\sqrt{2}$ is a CP-even (odd) state, which is identified by its CP-even (odd) decay products. For instance, the states $K_S\pi^0$, $K_S\rho^0$, $K_S\omega$, $K_S\phi$ identify a D_2^0, while $\pi^+\pi^-$, K^+K^- represent a D_1^0. For $D_1^0 = (D^0 + \overline{D}^0)/\sqrt{2}$ we have

$$
\begin{aligned}
\sqrt{2}A(B^+ \to D_1^0 K^+) &= A(B^+ \to D^0 K^+) + A(B^+ \to \overline{D}^0 K^+), \\
\sqrt{2}A(B^- \to D_1^0 K^-) &= A(B^- \to \overline{D}^0 K^-) + A(B^- \to D^0 K^-).
\end{aligned}
\tag{22}
$$

D^0 and $\overline{D}^0$, states of specific flavor, are identified by the charge of the decay lepton or kaon. The amplitudes of $B^+ \to D^0 K^+$ and $B^+ \to \overline{D}^0 K^+$ are shown schematically in Fig. 3. Their CKM factors, $V_{ub}^* V_{cs}$ and $V_{cb}^* V_{us}$, are of comparable magnitude. Their weak phases are γ and zero, respectively. The two relations (22) are described by the two triangles in Fig. 4 representing the B^+ and B^- decay amplitudes.

γ is determined with a two-fold ambiguity by the rates of the above six proccesses, two pairs of which are equal. Note that γ can be measured also when $\delta_1 - \delta_2 = 0$, in which case no asymmetry is observed and the two triangles in Fig. 4 must be drawn up-side-down with respect to each other. The feasibility of observing a CP asymmetry in $B^+ \to D_{1(2)}^0 K^+$ and the precision of measuring γ depend on the branching ratios of the three related decay processes, and on the values of the weak and strong phases. The decay $B^+ \to D^0 K^+$ is expected to be color-suppressed in addition to being CKM suppressed. Using a value of 5×10^{-6} for the branching ratio of this process, the feasibility for observing a CP asymmetry in $B^+ \to D_{1(2)}^0 K^+$ was studied [19] as function of γ and $\delta_2 - \delta_1$, for a (symmetric) $e^+e^- \to \Upsilon(4S)$ B-factory with an integrated luminosity of $20fb^{-1}$. The discovery region was found to cover a significant part of the $(\gamma, \delta_2 - \delta_1)$ plane. For small final state phase differences this experiment is sensitive mainly to values of γ around 90^0.

Large values of $\delta_2 - \delta_1$ allow a useful measurement of γ in the range $50^0 \leq \gamma \leq 130^0$. This method was generalized to quasi-two-body decays $B \to DK_i \to DK\pi$, where K_i are excited kaon resonance states with masses around 1400 MeV [20]. These resonances give rise to large calculable final state phases and thus may enhance the asymmetry.

4.3 Using SU(3) to determine γ from $B^+ \to \pi\pi$, πK, ηK [21][22]

Flavor SU(3) symmetry can be used to obtain relations among a variety of B decays [23]. Recently we have applied this symmetry and linearly broken SU(3) to two body decays [24]. We neglected small annihilation-like contributions which are expected to be suppressed by f_B/m_B. This suppression can be tested, for instance, by pushing upper limits on $B^0 \to K^+K^-$ down to a level of 10^{-7}. In the following discussion we use quark diagrams as a particularly useful representation of SU(3) amplitudes.

Consider the decay $B^+ \to \pi^0 K^+$ to which the two diagrams of Fig. 5 contribute. These diagrams represent a strong penguin and a tree amplitude, and are related by symmetry to measurable amplitudes of other decay processes. The penguin amplitude is related by isospin (exchanging $u\bar{u}$ by $d\bar{d}$) to the amplitude of $B^+ \to \pi^+ K^0$. The annihilation contribution to the latter process is neglected. The tree amplitude is related by SU(3) (exchanging $\bar{s}$ by $\bar{d}$) to the amplitude of $B^+ \to \pi^+\pi^0$, which receives no strong penguin contribution. SU(3) breaking can be introduced into this relation by assuming factorization of tree amplitudes. Thus the tree amplitude is given by $(f_K/f_\pi)|V_{us}/V_{ud}|A(\pi^+\pi^0)$. One therefore obtains a simple triangle relation between the three B^+ decay amplitudes:

$$A(\pi^0 K^+) - \frac{1}{\sqrt{2}} A(\pi^+ K^0) = \frac{f_K}{f_\pi} |\frac{V_{us}}{V_{ud}}| A(\pi^+\pi^0) \ , \tag{23}$$

and a similar relation holds among the corresponding B^- decay amplitudes. These two relations are analogous to Eqs.(22). They are described by two triangles very similar to those of Fig. 4. In the present case the two triangles share a common base given by $A(\pi^- \overline{K}^0) = A(\pi^+ K^0)$, and the angle between the sides describing $A(\pi^-\pi^0)$ and $A(\pi^+\pi^0)$ is 2γ. Measurements of the four rates into $\pi^0 K^+, \pi^0 K^-, \pi^+ K^0, \pi^+\pi^0$, suffices to determine γ.

This method is not as clean as the one using $B^\pm \to D^0 K^\pm$ decays. Contributions from electroweak penguin diagrams, which do not obey the above isospin relation, were neglected. Such terms do not cancel on the left-hand-side of (23). Although these terms are suppressed by a factor of about λ (as in (18)) relative to strong penguin terms which dominate the two amplitudes of $B^+ \to \pi K$, they spoil eq.(23) which relates the difference of these two amplitudes to the CKM-suppressed amplitude on the right-hand-side.

One way to recover an SU(3) triangle relation, with the inclusion of electroweak pen-

guin terms, is to use final states which involve the octet component of the η. One finds:

$$A(\pi^0 K^+) + \sqrt{2} A(\pi^+ K^0) = \sqrt{3} A(\eta_8 K^+) . \tag{24}$$

These amplitudes and their corresponding charge-conjugates can be related to $A(\pi^+\pi^0)$ as shown in Fig. 6, which could in principle determine γ. The problem here is that one would have to extract the amplitude into η_8 from B decay measurements into states involving η and η'. In addition there are corrections to (24) from SU(3) breaking terms.

Another way to resolve the uncertainty due to electroweak penguin terms is to use, in addition to $B \to \pi K$ decays also $B_s \to \pi^0 \eta_8$. These amplitudes obey a quadrangle relation, from which γ may in principle be determined. The problem here is that, while $B \to \pi K$ are expected to have branching ratios of about 10^{-5}, the branching ratio of the electroweak penguin dominated $B_s \to \pi^0 \eta_8$ decay is estimated to be less than 10^{-6}.

4.4 Large final state phases from interference between resonance and background [25]

CP asymmetries in charged B decays are prportional to a sine of the final-state phase-difference of two interfering amplitudes (see (21)). So far there exists no experimental evidence for final state phases in B decays, and it has been often assumed that such phases are likely to be small in decays to two light high momentum particles. This would lead to small asymmetries. Evidence for strong phases, related to final states with well-defined isospin and angular momentum, can be obtained from $B \to \overline{D}\pi$ decays. The amplitudes into $D^-\pi^+$, $\overline{D}^0\pi^0$, $\overline{D}^0\pi^+$ obey a triangle relation, from which the phase-difference between the $I = 1/2$ and $I = 3/2$ amplitudes may be determined. The present branching ratios of these decays already imply an upper limit [26], $\delta_{1/2} - \delta_{3/2} < 35°$. Improved measurements of these braching ratios may lead to more stringent bounds. Assuming, as may turn out to be the case, that final state phase differences are small in cases of interest, one would be looking for particular circumstances in which these phases are enhanced. Here we wish to demonstrate one such case.

Consider the decay $B^+ \to \chi_{c0}\pi^+$, $\chi_{c0} \to \pi^+\pi^-$, where one is looking for a final state with three pions, two of which have an invariant mass around $m(\chi_{c0}) = 3415$ MeV. The width of this $J^P = 0^+$ $c\bar{c}$ state, $\Gamma(\chi_{c0}) = 14 \pm 5$ MeV, is sufficiently large to provide a large, and in fact maximal, CP conserving phase. The decay amplitude into three pions, where two pions are at the resonance, consists of two terms with different CKM phases (we neglect a small penguin term):

R= a resonating amplitude, consisting of a product of the weak decay amplitude of $B^+ \to \chi_{c0}\pi^+$ involving a real CKM factor $V_{cb}^* V_{cd}$ (a_w=real), the strong decay amplitude of $\chi_{c0} \to \pi^+\pi^-$ (a_s=real), and a Breit-Wigner term for the intermediate χ_{c0}.

D= a direct decay amplitude of $B^+ \to \pi^+\pi^-\pi^+$ involving a CKM factor $V_{ub}^* V_{ud}$ with phase γ, which we write as $(d/m_B)\exp(i\gamma)$ (d=real).

$$R = a_w a_s \frac{\sqrt{m\Gamma}}{s - m^2 + im\Gamma} \quad , \qquad D = \frac{d}{m_B} \exp(i\gamma) \ . \tag{25}$$

The total amplitude is $R + D$.

The $B^+ - B^-$ decay rate asymmetry, integrated symmetrically around the resonance, is given by

$$Asym. \approx -2\frac{d}{a_w a_s}\frac{\sqrt{m\Gamma}}{m_B} \sin\gamma \ . \tag{26}$$

This is a special case of (21), in which the strong phase difference between the resonating and direct amplitudes is maximal, $\delta_D - \delta_R = \pi/2$. Note that all strong phases other than due to the resonance width were neglected. The asymmetry can be expressed in terms of the corresponding branching ratios

$$Asym. \approx f(0)\sqrt{\frac{B(B^+ \to \pi^+\pi^-\pi^+)_{nonres.}}{B(B^+ \to \chi_{c0}\pi^+)B(\chi_{c0} \to \pi^+\pi^-)}}\frac{\sqrt{8\pi m\Gamma}}{m_B} \sin\gamma \ . \tag{27}$$

$f(0)$ is the fraction of nonresonating three pion events, where the two pions at the resonance mass carry zero angular momentum. (Only this part of the direct amplitude interferes with the resonance amplitude). Model-dependent calculations show that $f(0)$ is of order one [27]. Using $B(\chi_{c0} \to \pi^+\pi^-) = 8 \times 10^{-3}$ [5], and taking reasonable estimates for the yet unmeasured branching ratios, $B(B^+ \to \pi^+\pi^-\pi^+)_{nonres.} \sim 10^{-5}$, $B(B^+ \to \chi_{c0}\pi^+) =$ a few $\times 10^{-5}$, one finds an asymmetry

$$Asym. = \mathcal{O}(1) \sin\gamma \ . \tag{28}$$

An observation of such a large asymmetry requires 10^8 or at most 10^9 B mesons.

5. Conclusion

An observation of CP violation outside the K meson system is extremely important by itself. B decays offer a large variety of such measurements which can test the CKM origin of CP violation. Certain CP asymmetries in B decays determine CKM phases in manners which are free of hadronic uncertainties. This will evidently provide a more precise determination than available today for these fundamental parameters. Consistency between these measurements and other (mostly CP conserving) measurements of CKM elements would confirm the CKM mechanism of CP violation. Inconsistencies, on the other hand, may lead the way to extensions of the Standard Model. This is a rich field which is constantly evolving, with new ideas being developed both on the theoretical and experimental frontiers. We should be able to enjoy its fruits by the end of this millennium.

Acknowledgements

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Figure captions

Fig. 1: The CKM unitarity triangle

Fig. 2: Isospin triangles of $B \to \pi\pi$, including electroweak penguin corrections

Fig. 3: Diagrams describing $B^+ \to \overline{D}^0 K^+$ and $B^+ \to D^0 K^+$

Fig. 4: Triangles describing Eqs.(22)

Fig. 5: Penguin and tree diagrams of $B^+ \to \pi^0 K^+$

Fig. 6: Triangles describing Eq.(24), its charge-conjugate and a relation with $A(\pi^+\pi^0)$

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

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